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AN EXTENSION OF THE ELECTROMECHANICAL ANALOGY IN THE DOMAIN OF HYDROSTATIC TRANSMISSIONS

Part I. THE ELECTROMAGNETIC AND ELECTROMECHANICAL ANALOGIES

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ABSTRACT. *The paper aims to expand the electromechanical analogy in other domains of technology: hydraulic, pneumatic, acoustic, sonic, and even in thermodynamics.*

In addition to the similarity of the equations and mathematical models, in the domain of fluidic systems we have highlighted the analogy of the circuit elements and some basic structures, for which the equivalent schemes are given. Analogy tables are presented, including the important sizes, units, symbols and generalized mathematical models applicable in all domains above and the advantages of the analogy and its limits of application are highlighted.

KEYWORDS. *Electromechanic / hydraulic analogy, generalized parameters, electric / mechanical / hydraulic resistance / inductance / capacity / impedance, analogy of sizes / equations, limits of the analogy.*

NOMENCLATURE

Symbol	Description
$\vec{A}$; $\vec{B} = \nabla \times \vec{A}$	electrodynamic vector potential
B_e $[\Omega^{-1}]$	electric susceptance
B_L $[N/m^2]$	bulk modulus of elasticity of liquid
$C_e = S_e^{-1}$ $[F]$	electric capacity / elastance
$f\left[Hz\right]$	circular frequency
$I_H = \Delta H / L$ $[-]$	hydraulic slope
K_D	diffusion coefficient in electrochemistry
K_H $[m^3]$	flow module in hydrodynamics
L_e $[H]$	electric inductance
M_H $[s^2 m^{-5}]$	hydraulic module in hydrodynamics
$n\left[-\right]$	nondimensional exponent of R_H - hydraulic resistance to motion
Q ; $\Delta Q [m^3/s]$	volumetric flow/leakage flow
$p; \Delta p = p_i - p_e$	pressure; drop/jump pressure
$R_e = G_e^{-1}$ $[\Omega]$	electric resistance / conductance
$\mathrm{Re} = v d \nu^{-1} \left[-\right]$	Reynolds number in hydraulic pipes
$R_L; R'[Ns/m^5$	linear/linearised hydraulic resistance to fluid motion
$R_N \left[\dfrac{Ns^n}{m^{3n+2}}\right]$	non-linear hydraulic resistance to motion of fluid
s $[m]$	curvilinear coordinate; string deviation
T $[K]$	thermodynamic temperature
U $[Nm]$	scalar potential of the force
$V_g [m^3/rot]$	Basic geometrical volume of rotary hydraulic volumic machines
$V, \Delta V$ $[m^3]$	volume; finite fluid volume variation
$\omega; \omega_0 \left[rad/s\right]$	angular/natural frequency
$z = x + jy$	complex number $(j = \sqrt{-1})$
$(R; \theta; \lambda)$	spatial polar coordinates
τ	apparent power mass density

ABBREVIATIONS

P.C.	particular case
B.C.; I.C.	boundary conditions/initial conditions
S.C.P.; S.D.P.	lumped/distributed parameters system
E.S.F.; M.S.F.	electric/magnetic quasi-stationary field
L.F.; I.F.	local (differential)form / integral form
O.D.E.; P.D.E.	ordinary/partial differential equations
E.Q.S.;M.Q.S.	electric/magnetic quasistationary

1. INTRODUCTION

The analogy is one of the basic methods used to research various areas of physics-specific phenomena, being based on the formal identity of mathematical models that describe the original phenomenon and the similar phenomenon. Therefore, the analogy method allows solving problems in a particular domain, using the results from another field, where the phenomena of the two domains have the same mathematical model (are analogue).

The scale of analogy is the constant ratio between the values of the two similar sizes, which belong to the two domains and must verify the (formally identical) mathematical models which express the conduct of the two phenomena. The analogy is partial, if only some of the quantities involved in the description of phenomena are analogue. During the dynamic modeling of complex systems, difficulties arise, regarding their direct analysis, so that sometimes the application of methods of study and indirect analysis are required, allowing the complete knowledge of the studied phenomenon, based on observations and experimental research performed on similar models.

The biunivocal correspondence between the original phenomenon and its model allows that, on the basis of rules and assumptions established a priori, the variables that can not be assessed on the primary (original) system can be determined on the analog model; based on information obtained from the model, some conclusions can be drawn on the original behavior. Frequently, the original system (a mechanical, acoustic, hydraulic, pneumatic or thermal system) is studied on an analog electric model, this providing the research of the phenomenon on an equivalent electric or electronic circuit, which allows the processing of results and the implementation of solving methods from the electric field in the area of interest. Similar models in the electric field are preferred, as their structure and operation have been improved based on results obtained in the theory of electrical circuits. In these circumstances, the analogy can be a process of synthesis of complex non-electric circuits, based on synthesis algorithms specific to electric or electronic circuits.

A detailed approach of the studies regarding the analogy is shown by Olson (1958), Kinsler (1962), Levi (1966), Hackenschmidt (1972), Stanomir (1989), and Fransua (1999).

2. REQUIREMENTS OF THE ELECTROMECHANICAL ANALOGY

The electromechanic analogy has the advantage that it can easily adapt to the following requirements specific to the study of physical phenomena by modeling (as shown by Stanomir, 1982).

1. The model should ensure a broad representation of the original, that is to allow highlighting its unknown properties; they must be better known than the ones of the original, or must be easily experimentally modeled;
2. The bonds of the non-electric system must be holonome, scleronome and linear and the system must have a linear equivalent graph and only elementary one-port structures.
3. The specific dynamic process of the original system to be studied on the analog model will strictly collapse (limit) to the domain of interest;
4. Based on the scheme of the original model, we can establish the equivalent electrical scheme; for example, the series / parallel connection of elements that goes into the original, must be replaced with similar elements of circuit, connected properly, in series / parallel;
5. The physical character of a quantity of the original system must be maintained to the study on the analogue system; for example, the hydraulic potential must have as analogous the electric potential, etc.
6. Any analogy must respect the principle of conservation of energy, so that the condition of compatibility from an analogy to the other derived from it, is to be a biunique correspondence of powers for the two domains of the similar phenomena; for this reason, the equations from the mathematical model of the analog electrical circuit must be isomorphic with the mathematical model equations of the studied (original) system, as defined in the biunique correspondence.

The analogy of non-electrical elements and circuits (mechanic, acoustic, thermic, hydraulic, pneumatic, sonic etc.) with the electrical ones allows the modeling of the original system, based on some active circuit elements (sources) connected to passive elements (resistances, inductances, capacities, perditances) through specific junction elements. Since the analogy of sizes and mathematical models also aims an analogy of the physical laws specific to analogue circuits (systems), some theorems and laws of the electric disciplines can be properly translated into laws and theorems expressing the phenomena of the domain of interest (the original domain); example: Kirchhoff's theorems, Ohm's law, the law of electromagnetic induction, the transfiguration theorem, superposition theorem, $Y \leftrightarrow \Delta$ transfiguration etc. (as shown by Mocanu, 1979, Şora, 1982, Stanomir, 1989).

3. GENERALIZED PARAMETERS

The need to extend the electromechanical analogy to other non-electric domains, which derive from mechanics, becomes obvious with the approach of interdisciplinary researches with applicability in science vanguard fields like space flight engineering, mecatronics etc. There is also a need to establish a common language based on the essential role of the extended analogy between the domains corresponding to physical systems that aim the interdisciplinary science objectives.

The degree of generality of the analogy can be significantly increased if the quantities involved in the description of the phenomena are brought to dimensionless forms and if a common language, with general validity is adopted. In order to highlight the domain of application for some extended analogies, generalized power variables are introduced: *f* (flow) and *e* (effort), and on this basis we can define the generalized energy variables (as shown by Buculei, 1993, Rădulescu, 2005).

We can generalize some non-electric sizes, similar to those associated with passive circuit elements (resistances, inductances, capacities, perditances etc.); these quantities are introduced taking into account the analogy of some basic laws from the disciplines with non-electric profile, with laws and equations of circuit and electrical machines theory. Table 1 gives a generalization of the basic parameters that characterize some systems/devices and the elements of machines and circuits.

Table 1. Generalized parameters

	Parameters
General	Displacement: $q(t)=\int_0^t f(\tau)d\tau,\quad q(0)=0$ Pulse: $\Im(t)=\int_e^t e(\tau)d\tau,\quad \Im(0)=0$ Generalized power: $P=ef=\Pi\ [W]$; Generalized energy: $E(t)=\int_0^t P(\tau)d\tau,\ E(0)=0\,[J]$ Action: $A(t)=q(t)\Im(t)\ [J]$; Impedance: $\underline{Z}=\underline{e}/\underline{f}=R+jX=\underline{Y^{-1}}[\Omega]$; $j=\sqrt{-1}$ Admittance: $\underline{Y}=G-jB\left[S=\Omega^{-1}\right]$
Mechanical	Displacement: $q_m=\begin{cases}x[m], \textit{stroke length} - \textit{for linear hydraulic volumic machines } (L.M.)\\ \varphi[rad], \textit{angle} - \textit{for rotary hydraulic volumic machines } (R.M.)\end{cases}$ Velocity: $w=\begin{cases}v=\dot{q}[m/s], \textit{linear velocity of the mobile equipment of the machine} - L.M.\\ \omega=\dot{\varphi}[rad/s], \textit{angular velocity of the rotative parts of the machine} - R.M.\end{cases}$ Effort: $\phi=\begin{cases}F[N], \textit{force reduced to the stroke of the machine} - L.M.\\ M[Nm], \textit{torque reduced to the rotor axis of the machine} - R.M.\end{cases}$ Mass: $\mathrm{M}=\begin{cases}m[kg], \textit{mass reduced to the stroke axis} - L.M.\\ J\left[Nm^2s\right], \textit{momentum of gyration reduced to the rotor axis} - R.M.\end{cases}$ Damping constant : $\underset{(R_m)}{k_w}=\begin{cases}k_v[Ns/m] - \textit{viscous friction constant} - L.M.\\ k_\omega[Nms] - \textit{viscous friction constant} - R.M.\end{cases}$ Mechanical stiffness: $k_q=\dfrac{d\phi}{dq}=\begin{cases}k_x[N/m], \textit{elastic constant, for linear deformations} - L.M.\\ k_\varphi[Nm/rad], \textit{elastic constant, for angular deformations} - R.M.\end{cases}$ Elementary work: $\delta L=\phi\cdot\delta q[J]$; Overall energy: $W=W_c+W_p=\frac{1}{2}\left(\mathrm{M}\ w^2+K_m q^2\right)\ [J]$
Hydraulic	Geometric capacity: $K=\begin{cases}V_g/2\pi\left[m^3/rad\right], V_g - \textit{basic geometric volume} - R.M.\\ A_1 \textit{ or } A_2\left[m^3/m\right], \textit{the piston area of the active chamber} - L.M.\end{cases}$ Hydraulic resistance to motion (friction): $R_H=\dfrac{d(\Delta p)}{\Delta Q}=\begin{cases}R_L\,[Ns/m^5], \textit{linear resistance}(\mathrm{Re}<\mathrm{Re}_{cr})\\ R_N[Ns^n/m^{3n+2}]; n\in(1;2], \textit{nonlinear resistance}(\mathrm{Re}>\mathrm{Re}_{cr})\end{cases}$ Hydraulic leakage conductance: $G_H=\Delta Q/p\left[m^5/N\right]$ Hydraulic resistance to acceleration (hydraulic inertance/inductance): $L_H=\Delta p/\dot{Q}=H\left[Ns^2/m^5\right]$; Hydraulic mobility: $M_H=L_H^{-1}\left[m^5/Ns^2\right]$ Hydraulic capacity: $C_H=\Delta Q/\dot{p}\left[m^5/N\right]$; Hydraulic resistance to deformation / hydraulic capacity: $D_H=C_H^{-1}\ \left[N/m^5\right]$ Hydraulic stiffness: $R_h=\dfrac{\partial\phi}{\partial q}\Big\vert_{Q=0}\left[Nm\cdot rad^{-2}\right]$ Hydraulic compliance: $\Lambda_h=C_H/K^2=R_h^{-1}\ [N^{-1}m^{-1}rad]$

4. THE ELECTRIC / MAGNETIC ANALOGY

Table 2 shows the analogy between the equations of the electric steady field and the ones of the magnetic field. Most of the sizes and definition relationships have the same shape in both domains (volumic density of the energy and forces, stored energy, flux / circulation of the fields, generalized forces etc.). Most laws and theorems are also similar (Kirchhoff's theorems, the constitutive relationships, the uniqueness and superposition theorems etc.), as shown by Levi (1966) and Şora (1982).

Table 3 presents the summarized analogy by comparing the definitions and properties of the electrostatic, electrokinetic and magnetic fields. For particular cases (P.C.) we have assumed the three mediums are linear, isotropic and homogenous. In this case there is also a similarity between some definitions and laws of the three domains (as mentioned by Stan, 2005).

The analogy between the stationary electric and magnetic fields can help solving some theoretical and experimental problems regarding the study of electric circuits using already known results in electrostatics / magnetostatics or vice versa. A good example would be determining some electrostatic characteristics using experimental models in electro-kinetics using electrolytic tanks or electroconductive paper. When choosing the physical model one must take into consideration the condition that the two analogue models have similar configurations.

The previous observations mentioned prove their utility mainly in the didactic field and mostly in interdisciplinary practical courses (electromechanics, magnetohydrodynamics, mecatronics, robotics etc.

5. THE ELECTROMECHANICAL ANALOGY

The electromechanical analogy (E.M.A.) is used in the study of simple oscillatory electric systems, on the basis of some models of elementary mechanic systems and in some cases the more complex discrete mechanic systems are analyzed using analogue electrical networks, considering the formal equivalence between Lagrange's equations and Kirchhoff's laws, as well as the practical possibilities of measurement of the state parameters of electrical circuits. In table 4 is presented the E.M.A. of the basic sizes for the I and II - type analogies.

In the E.M.A., the ideal passive elements that form a mechanical lumped-parameter (discrete) system are represented using analogue symbols of the elements *R*, *L*, *C* specific to electrical lumped-parameter circuits as mentioned by Nedelcu (1978), Iacob (1980) and Stanomir (1989).

The ideal active elements (the mechanical sources) are adopted in analogy to the sources specific to electric circuits and are associated a positive sense and polarities which correspond to the positive sign of the mechanical power in the circuit (as shown by Fransua, 1999).

On the basis of this rationing, the ideal passive elements of the mechanical lumped-parameter system, noted R_m, L_m, C_m, as well as the sources of generalized force (Φ_m) or velocity (w_m) are represented using symbols presented in the second part of this paper (as mentioned by Fransua, 1999 and Radulescu, 2005).

When adopting the mechanical scheme one must have in mind the compliance between d'Alembert's principle and Kirchhoff's first law, as well as the correspondence between the passive elements of the two analogue circuits. When drafting the analogue scheme for the circuit corresponding to a mechanic system one can distinguish the following stages:

1. The compounding bodies are reduces to material points which correspond to nodes of mechanic network with the velocity in ratio to a standard branch point;
2. The elements of circuit are inserted between the nodes;
3. The equations of equilibrium of forces (ϕ_{m_k}) are written for every node;
4. The mathematical model is solved (the integral - differential equations) taking into consideration the initial conditions and the velocity of the node w_j is obtained.

The electromechanical analogy allows solving issues related to vibrations of complex mechanical systems by replacing them with equivalent electrical network, which can be easily studied on the basis of "standard" results obtained in the theory of electrical circuits.

Using this analogy is only possible if the mechanical system studied is linear, if its vibration amplitudes are small enough. It is essential to obtain the correct configuration of the electrical scheme of the vibrating mechanical system, given the following recommendations (as shown by Fransua, 1999; Stanomir, 1989):

- for the analogy of impedances, to a mechanical series assembly corresponds a parallel electrical circuit and vice versa.
- for the analogy in admittances, to a mechanical parallel assembly corresponds a parallel electric circuit and to a mechanic series assembly corresponds a series electric circuit.

Usually, the equivalent electromechanical scheme is determined, which corresponds to the mathematical model, based on which one can determine the mechanical impedance or mechanical mobility.

For example, Fig. 1 presents an elementary mechanical system (autovehicle and trailer) in translation movement (a) and its analogue mechanical scheme (b).

Table 2. The electric - magnetic analogy of parameters

Electrics	Denomination	Magnetism
$\vec{E}(\vec{r};t)$ $[V/m]$	Strength of the vectorial field	$\vec{H}(\vec{r};t)$ $[A/m]$
$\vec{D}(\vec{r};t)$ $[C/m^2]$	Flux density	$\vec{B}(\vec{r};t)$ $[T]$
$\vec{p}_e[C\cdot m]$	Moment	$\vec{p}_m[Nm/T]$
$\vec{P}_p = d\vec{p}_e / dV$	Polarization / magnetization	$\vec{M} = d\vec{p}_m / dV$
$\vec{C}_e = \vec{p}_e \times \vec{E}$	Torque	$C_m = \vec{p}_m \times \vec{B}$
$\vec{F}_e = \nabla(\vec{p}_e \cdot \vec{E})$	Force	$\vec{F}_m = \nabla(\vec{p}_m \cdot \vec{B})$
$\vec{f}_e = \rho_V \vec{E} - (1/2)E^2 \nabla\varepsilon$ $(\varepsilon'(\tau)=0)$	Volumetric force	$\vec{f}_m = \vec{J} \times \vec{B} - (1/2)H^2 \Delta\mu$ $(\mu'(\tau)=0)$
$X_k = -\dfrac{\partial W_e(q_0;x)}{\partial x_k}$	Theorem of generalized forces	$X_k = -\dfrac{\partial W_m(\varphi;x)}{\partial x_k}$
$\vec{T}_n = \vec{E}(\vec{D}\cdot\vec{n}) - \frac{1}{2}(\vec{D}\cdot\vec{E})\vec{n}$ $\vec{T}_n = \vec{n}\cdot \overset{\Rightarrow}{T}_e$	Maxwellian tensions	$\vec{T}_n = \vec{H}(\vec{B}\cdot\vec{n}) - \frac{1}{2}(\vec{H}\cdot\vec{B})\vec{n}$ $\vec{T}_n = \vec{n}\cdot \overset{\Rightarrow}{T}_m$
$\varepsilon[F/m]$	Vacuum permittivity/ permeability	$\mu_0[H/m]$
χ_e	Susceptivity	χ_m
$\varepsilon_r = 1 + \chi_e$	Relative permittivity/ permeability	$\mu_r = 1 + \chi_m$
$\varepsilon_{diff} = \varepsilon_0^{-1} D'(E)$	Differential permittivity/ permeability	$\mu_{diff} = \mu_0^{-1} B'(H)$
$\vec{P}_t = \varepsilon_0 \overset{\Rightarrow}{X}_e \cdot \vec{E}$	Law of polarization/ magnetization	$\vec{M}_t = \overset{\Rightarrow}{X}_m \cdot \vec{H}$
$\vec{P}_p$	Permanent polarization/ magnetization	$\vec{M}_p$
$u_{em} = \oint_{(C)} \vec{E}\cdot d\vec{r}$	Electro / magnetomotive force	$u_{mm} = \oint_{(C)} \vec{H}\cdot d\vec{r}$
$\psi = \iint_{(\Sigma)} \vec{D}\cdot d\vec{\sigma}$	Flux	$\phi = \iint_{(\Sigma)} \vec{B}\cdot d\vec{\sigma}$
$\psi^* = \iint_{(\Sigma)} \vec{E}\cdot d\vec{\sigma}$	Flux of the field ($\varepsilon = ct.$)	$\phi^* = \iint_{(\Sigma)} \vec{H}\cdot d\vec{\sigma}$
$V_e = V_e(\vec{r};t)$	Scalar potential	$V_m = V_m(\vec{r};t)$
$\nabla\times\vec{E} = \vec{0}$; $\vec{E} = -\nabla V_e$	Irotational field	$\nabla\times\vec{H} = \vec{0}$; $\vec{H} = -\nabla V_m$
$u_e = V_{e_1} - V_{e_2}$	Voltage / magnetic tension	$u_m = V_{m_1} - V_{m_2}$
$\nabla\cdot\vec{D} = 0$ $(q_e = 0)$	Solenoid field	$\nabla\cdot\vec{B} = 0$
$w_e = \frac{1}{2}\vec{E}\cdot\vec{D}$	Volumetric density of the energy	$w_m = \frac{1}{2}\vec{H}\cdot\vec{B}$
$W_e = \frac{1}{2} q_e V_e$	Energy in the capacitor/ inductor	$W_m = \frac{1}{2}\phi i$
$\Delta V_e + \rho_V \varepsilon_0^{-1} = 0$; scalar: $\vec{E} = -\nabla V_e$	Poisson's equation Potential	$\Delta\vec{A} + \mu\vec{J} = 0$; vector: $\vec{B} = \nabla\times\vec{A}$
$\rho_V = 0 \Rightarrow \Delta V_e = 0$	P.C.: Laplace's equation	$\vec{J} = \vec{0} \Rightarrow \Delta\vec{A} = \vec{0}$

Fig. 2 shows an example of elementary mechanical system in rotary movement (gear box): the cinematic scheme (a) and the analogue calculus scheme (b).

The two schemes include specific notations for the two types of motion, but they can also be written in a generalized form, as shown in Table 1.

Table 3. The electrostatic-electrokinetic-magnetic analogy

Electrostatic field	Electrokinetic field	Magnetic field
Differential equation of the field lines		
$d\vec{r}\times\vec{E}=\vec{0}$	$d\vec{r}\times\vec{J}=\vec{0}$	$d\vec{r}\times\vec{H}=\vec{0}$
Characteristics of the material		
Electric permittivity $\varepsilon=\varepsilon_0\varepsilon_r$ $[F/m]$	Electric conductivity/resistivity $\sigma=\rho_e^{-1};\rho_e\overset{(not)}{=}\rho=\rho_0\rho_r$ $[\Omega m]$	Magnetic permeability/reluctivity $\mu=\mu_0\mu_r=\nu^{-1}$ $[H/m]$; ν - magnetic reluctivity
Global energetic relations		
Electrostatic energy $W_e=\frac{1}{2}\left(\iiint_{(D)}\rho_v V_e dV+\iint_{(\Sigma)}\rho_s V_e d\sigma\right)$	Law of degradation of electric energy $W_J=\iiint_{(D)}w_J dV$; $w_J=\int_0^t\vec{E}\cdot\vec{J}d\tau$ (Joule-Lenz)	Magnetic energy $W_m=\frac{1}{2}\left(\iiint_{(D)}\vec{A}\cdot\vec{J}dV+\iint_{(\Sigma)}\nabla\cdot\left(\vec{A}\times\vec{H}\right)d\sigma\right)$
Particular forms of the Maxwell's equations		
Constitutive relations $\vec{D}=\overset{\Rightarrow}{\varepsilon}\,\vec{E}+\vec{P}_p$ PC: $\overset{\Rightarrow}{\varepsilon}\to\varepsilon=ct;\ \vec{P}_p=\vec{0}\Rightarrow\vec{D}=\varepsilon\vec{E}$	*The electric conduction law* $\vec{J}=\sigma\left(\vec{E}+\vec{E}_i\right)$ (Ohm's law) PC: $\vec{E}_i=0\Rightarrow\vec{J}=\sigma\vec{E}\Leftrightarrow\vec{E}=\rho\vec{J}$	*Constitutive relations* $\vec{B}=\overset{\Rightarrow}{\mu}\cdot\vec{H}+\mu_0\vec{M}_p$ PC: $\overset{\Rightarrow}{\mu}\to\mu=ct;\vec{M}_p=\vec{0}\Rightarrow\vec{B}=\mu\vec{H}$
Electric flux law (Gauss's law) (L.F.) $\nabla\cdot\vec{D}=\rho_v\Leftrightarrow$ $\Leftrightarrow\oiint_{(\Sigma)}\vec{D}\cdot d\vec{\sigma}=q_e$ (I.F.) P.C.: $\rho_v=0\Rightarrow\nabla\cdot\vec{D}=0(\text{L.F.})\Leftrightarrow$ $\Leftrightarrow\oiint_{(\Sigma)}\vec{D}\cdot d\vec{\sigma}=0$ (I.F.)	Law of electricity conservation (L.F.) $\nabla\cdot\vec{J}=-\dot{\rho}_v\Leftrightarrow$ $\oiint_{(\Sigma)}\vec{J}\cdot d\vec{\sigma}=-\dot{q}_e(t)$ (I.F.) P.C.: $\dot{\rho}_v=0\Rightarrow\nabla\cdot\vec{J}=0(\text{L.F.})\Leftrightarrow$ $\Leftrightarrow\oiint_{(\Sigma)}\vec{J}\cdot d\vec{\sigma}=0$ (I.F.)	Magnetic flux law (Gauss's law) (L.F.) $\nabla\cdot\vec{B}=0\Leftrightarrow$ $\Leftrightarrow\oiint_{(\Sigma)}\vec{B}\cdot d\vec{\sigma}=0$ (I.F.)
P.C.: the flux of the field vector through one field tube (stationary field)		
$\iint_{(\Sigma)}\vec{D}\cdot d\vec{\sigma}=\Psi$	$\iint_{(\Sigma)}\vec{J}\cdot d\vec{\sigma}=i$	$\iint_{(\Sigma)}\vec{B}\cdot d\vec{\sigma}=\Phi$
Law of electrostatic potential (V_e); irrotational field: $\nabla\times\vec{E}=\vec{0}\Rightarrow\ \vec{E}=-\nabla V_e(\text{L.F.})\Leftrightarrow$ $\Leftrightarrow\int\vec{E}\cdot d\vec{r}=V_{e_1}-V_{e_2}$ (I.F.)	E.Q.S. field: *Faraday's law* $(\vec{v}=0)$ $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}(\text{L.F.})\Leftrightarrow$ $\Leftrightarrow\oint\vec{E}\cdot d\vec{r}=-\frac{\partial\Phi}{\partial t}$ (I.F.) PC: $\dot{\vec{B}}=\vec{0}\Rightarrow\vec{E}=-\nabla V_e$; V_e - scalar electric potential	M.Q.S. field: *law of magnetic circuits* (*Ampere's law*) $\nabla\times\vec{H}=\vec{J}$ (D.F.) $\Leftrightarrow$ $\Leftrightarrow\oint_{(C)}\vec{H}\cdot d\vec{r}=\iint_{(\Sigma)}\vec{J}\cdot d\vec{\sigma}$ (I.F.) PC: $\vec{J}=0\Rightarrow\vec{H}=-\nabla V_m$; V_m - scalar magnetic potential
Usual circuit elements (stationary field)		
Electric capacitor: $C_e=\frac{q_e}{u_C};u_C=V_{e_1}-V_{e_2}$	Electric conductance: $G_e=\frac{i}{u_R}=R_e^{-1};u_R=\int_{(C)}\vec{E}\cdot d\vec{r}$	Magnetic permeance: $\Lambda_m=\frac{\Phi_m}{u_m}=R_m^{-1};u_m=\int_{(C)}\vec{H}\cdot d\vec{r}$
Electric elastance: $S_e=\int_0^l\frac{ds}{\varepsilon S}=C_e^{-1}\ \left[F^{-1}\right]$	Electric resistance: $R_e=\int_0^l\frac{ds}{\sigma S}=G_e^{-1}\ [\Omega]$	Magnetic reluctance: $R_m=\int_0^l\frac{ds}{\mu A}=\Lambda_m^{-1}\ \left[\frac{A}{Wb}\right]$

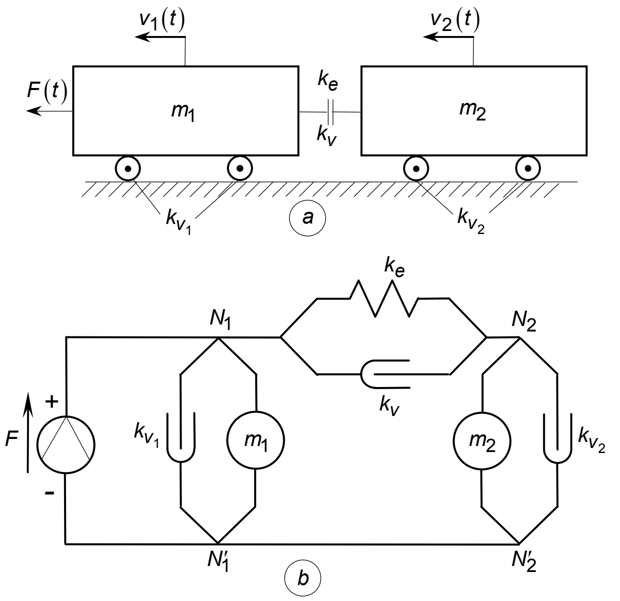


Fig. 1. Example of equivalent scheme for a mechanical system in translation movement

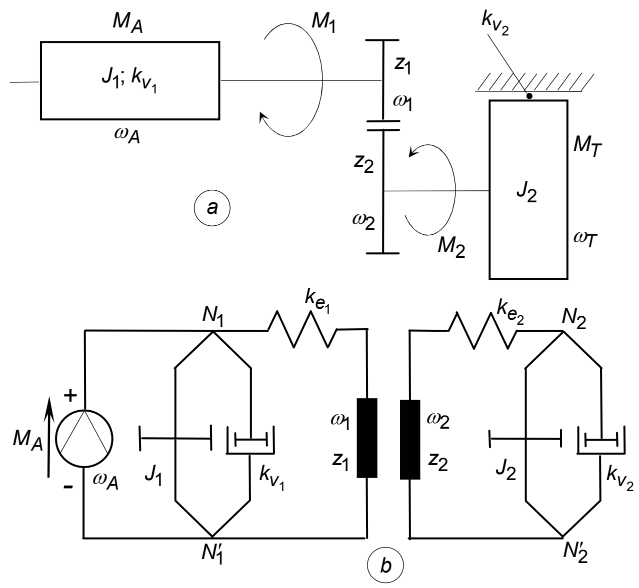


Fig. 2. Example of equivalent scheme for a mechanical system in rotation movement

When drafting these schemes, we have used the II - type analogy ($\phi_m \leftrightarrow i$; $w_m \leftrightarrow w_e$) which assures the compatibility of d'Alembert's principle and Kirchhoff's first law. If the I - type analogy would have been used ($\phi_m \leftrightarrow u_e$; $w_m \leftrightarrow i$), the two analogue circuits would have had different configurations, because d'Alembert's principle would have been incorrectly associated to Kirchhoff's second law.

Table 4. The electromechanic analogy of parameters

<table>
<tr><td colspan="2">Generalised parameters in mechanics</td><td colspan="4">Electric analogy</td></tr>
<tr><td colspan="2">I type (Z)</td><td colspan="4">II type (Y)</td></tr>
<tr><td colspan="2">Displacement: $q_m(t) = \int_0^t w_m(\tau)d\tau$</td><td colspan="2">Charge: $q_e(t) = -\int_0^t i(\tau)d\tau$</td><td colspan="2">Flux: $\Phi_e = -\int_0^t u_e(\tau)d\tau$</td></tr>
<tr><td colspan="2">Velocity: $w_m = \overset{\circ}{q}_m(t)$</td><td colspan="2">Current:</td><td colspan="2">Voltage: $u_e(t)$</td></tr>
<tr><td colspan="2">Acceleration: $\overset{\circ}{w}_m = \overset{\circ\circ}{q}_m$</td><td colspan="2">Velocity of i: $i(t)$</td><td colspan="2">Velocity of u_e: $\overset{\circ}{u}_e(t)$</td></tr>
<tr><td colspan="2">Mass: M</td><td colspan="2">Inductance: L_e</td><td colspan="2">Capacitance: C_e</td></tr>
<tr><td colspan="2">Stiffness: k_q</td><td colspan="2">Inverse of capacitance: C_e^{-1}</td><td colspan="2">Inverse of inductance: L_e^{-1}</td></tr>
<tr><td colspan="2">Damping coefficient: k_w</td><td colspan="2">Resistance: R_e</td><td colspan="2">Conductance: G_e</td></tr>
<tr><td rowspan="3">Effort</td><td>Inertial: $\phi_i(t) = \mathrm{M}\,\overset{\circ}{w}$</td><td rowspan="3">Voltage</td><td>Inductive: $u_L = L_e\overset{\circ}{i}(t)$</td><td rowspan="3">current</td><td>Capacitive: $i_c = C_e\overset{\circ}{u}_e(t)$</td></tr>
<tr><td>Elastic: $\phi_q = k_q q_m$</td><td>Capacitive: $u_C = C_e^{-1}q_e(t)$</td><td>Inductive: $i_L = L_e^{-1}\Phi_e$</td></tr>
<tr><td>Damping: $\phi_w = k_w w_m$</td><td>Resistive: $u_R = R_e i(t)$</td><td>Conductive: $i_G = G_e u_e$</td></tr>
<tr><td colspan="6">Irrotational field</td></tr>
<tr><td colspan="2">$\vec{\phi}_m = -\nabla U_m$</td><td colspan="2">$\vec{E} = -\nabla V_e$</td><td colspan="2">$\vec{H} = -\nabla V_m$</td></tr>
<tr><td colspan="6">Generalized force</td></tr>
<tr><td colspan="2">$X_k = \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_{m_k}}$</td><td colspan="2">$X_k = +\left(\frac{\partial W_e}{\partial x_k}\right)_{q_e}$</td><td colspan="2">$X_k = -\left(\frac{\partial W_m}{\partial x_k}\right)_{\Phi_e}$</td></tr>
<tr><td colspan="6">Stationary characteristic</td></tr>
<tr><td colspan="2">$\phi_m = \phi_m(w_m)$</td><td colspan="2">$u_e = u_e(i)$</td><td colspan="2">$u_m = u_m(\Phi_e)$</td></tr>
<tr><td colspan="2">Impulse of the body: $\vec{\Im}(t) = \mathrm{M}\,\vec{w}$</td><td colspan="2">Flux of the inductor: $\Phi_e = Li$</td><td colspan="2">Charge of the capacitor: $q_e = C_e u_e$</td></tr>
<tr><td colspan="2">Mechanic power: $P_m = \phi_m w_m$</td><td colspan="4">Electric power: $P_e = u_e i$</td></tr>
</table>

Table 4. Continuation

<table>
<tr><td colspan="2">Elementary mechanical work:
$\delta L_m = \phi_m \delta q_m$</td><td colspan="3">Elementary electric work
$\delta L_e = u_e \delta q_e$ | $\delta L_e = i_e \delta \phi_e$</td></tr>
<tr><td colspan="2">Mechanical reactance:
$X_m = \omega M - \omega^{-1} k_q$</td><td>Electric reactance:
$X_e = \omega L_e - \omega^{-1} C_e^{-1}$</td><td colspan="2">Susceptance: $B_e = \frac{X_e}{Z_e^2}$</td></tr>
<tr><td colspan="2">Mechanical impedance: $\underline{Z}_m = \frac{\phi_m}{\underline{w}_m}$
$\underline{Z}_m = k_w + jX_m$</td><td>Electric impedance: $\underline{Z}_e = \frac{U_e}{\underline{I}}$
$\underline{Z}_e = R_e + jX_e$</td><td colspan="2">Admittance: $\underline{Y}_e = \frac{\underline{I}}{\underline{U}_e}$
$\underline{Y}_e = G_e - jB_e$</td></tr>
<tr><td rowspan="3">Mechanic energy</td><td>Kinetic: $W_k = \frac{1}{2} M w_m^2$</td><td>Solenoid: $W_L = \frac{1}{2} L_e i^2$</td><td rowspan="3">Electric energy:</td><td>Capacitor: $W_C = \frac{1}{2}\frac{q_c^2}{C_e}$</td></tr>
<tr><td>Potential: $W_p = \frac{1}{2} k_q q_m^2$</td><td>Capacitor: $W_C = \frac{1}{2}\frac{q_e^2}{C_e}$</td><td>$W_L = \frac{1}{2} L_e i^2$</td></tr>
<tr><td>Dissipative: $W_d = \int_0^t k_w w^2 d\tau$</td><td>Resistance:
$W_J = \int_0^t R_e i^2(\tau) d\tau$</td><td>Conductance:
$W_J = \int_0^t G_e u_R^2(\tau) d\tau$</td></tr>
<tr><td colspan="5">Total energy</td></tr>
<tr><td colspan="2">$W_t = W_d + \frac{1}{2}\left(M w_m^2 + k_q q_m^2\right)$</td><td colspan="3">$W_t = W_J + \frac{1}{2}\left(L_e i^2 + \frac{q_e^2}{C_e}\right)$</td></tr>
<tr><td colspan="5">Action</td></tr>
<tr><td colspan="2">$\vec{A}_m = \Im(t) q_m(t)$</td><td colspan="3">$A_e = \Phi(t) q_e(t)$</td></tr>
</table>

Table 5 and Table 6 give two commonly used mathematical models with generalized (unified) notations corresponding to the previously defined generalized parameters. Table 5 shows the basic features of the linear oscillator with damping and harmonic excitation, seen as a model for the study of systems with concentrated parameters.

The mechanical system with concentrated parameters shows a basic structure which materializes the basic effects (inertial, elastic, dissipative) and is characterized by the constitutive equation that indicates the dependence between a kinematic size (displacement, velocity or acceleration) and an effort size (force or torque); based on this dependency we can identify the mechanical impedance.

The mechanical system with distributed parameters can be one-dimensional or two-dimensional, where waves arise (in particular small oscillations) for which the equations of the mathematical model are linear. The modeling of non-electric systems can be based on electric systems with distributed parameters whose mathematical model contains the telegraph, the second order differential equations with partial derivates which relate the parameters *u* (*t, x*) and *i* (*t, x*).

Table 6 shows the basic features of systems with distributed parameters, taking into account the two generalized basic sizes involved in relationships (1) and (2), intensive and complementary sizes: *e* (effort) and *f* (flow); these sizes can be identified in each of the areas subject to observation in this work (electric, namely, non-electric), so based on the mathematical model in which they occur, we can obtain the dynamic characteristics in the frequency domain, as mentioned by Olson (1958), Iacob (1980) and Stanomir (1989).

The mechanic phenomena specific to continuous material spectrums can be modeled on the basis of the equations of the electromagnetic field, using the calculus of variations and having in view the Lagrange density function and Hamilton's equations. Therefore, Table 6 summarizes the model with distributed parameters, which has a general character, allowing the study of analog phenomena in several fields (electrical, mechanical, hydraulic, acoustic, sonic, etc.).

Table 7 presents the study of a mechanical circuit (R_m, L_m, C_m) compared to the analogue electrical circuit; in both assemblies (series / parallel) are shown two types of analogies (in impedance - analogy type I) and (in admittance - analogy type II), based on the mathematical model in Table 5.

From the above it is confirmed that the base criterion for choosing the right type of analogy is the correspondence between Kirchhoff's theorems and the two base equations which interfere in the description of the analogue (non-mechanic) system. One can observe that the I-type analogy regards Kirchhoff's first theorem and the mechanics equation resulting from Newton's first law (the equilibrium of sizes *e*). The II-type analogy regards Kirchhoff's second theorem and the mechanics equation which results from Newton's second law (the balance of the sizes *f*). A particular case is the *electroacoustic analogy* (E.A.A.), which allows modeling the acoustic systems using elementary electric systems. For example, Fig. 3 shows the acoustic (a) and the analogue electric scheme (b) for a classic microphone (as shown by Iacob, 1980). The schemes have been adopted considering the **II** - type analogy ($u_e \leftrightarrow v$; $i \leftrightarrow p_a$) and the notations correspond to those in Table 8. The mathematical model of the scheme corresponds to an oscillatory system without losses, whose scheme is reduced to a series combination ($C_{e_{eq}} + L_e$) connected in parallel to C_{e_3}.

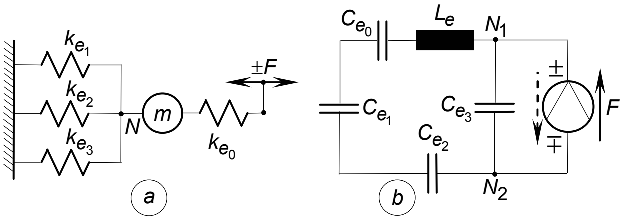


Fig. 3. The mechanical scheme and the analogue electric scheme of the microphone

The E.A.A. is frequently used in electroacoustics (as shown by Stanomir, 1989) where various inhomogeneous converting subsystems interfere, specific to unconventional conversion microdrives.

Table 8 presents the study of a section of acoustic line (pipe) with distributed parameters based on the mechanical-acoustic analogy (Type **I** - direct, respectively, type **II** - reverse), obtained from the reference mathematical model from Table 6.

For acoustic systems with distributed parameters the geometric dimensions of the system are comparable to the wavelength λ_u, for which there is a phase shift, due to propagation. When designing technical systems a mechanic-mechanic analogy is sometimes needed, being regarded in some cases also as a similitude. Next, we present an example of mechanic-mechanic analogy.

Table 5. The basic features of the linear oscillator (for S.C.P.)

Parameter	Definition and calculus equations	Other formulae, notations	Resonance values
Pulsation	Natural: $\omega_0 = (M_m C_m)^{-1/2} = \left(K_m / M_m\right)^{1/2}$; pseudo-pulsation: $\omega_p = \sqrt{\omega_0^2 - \delta^2} = \omega_0\sqrt{1-\zeta^2}$		
Mechanical resistance (damping factor)	R_m	Damping factor: $\zeta = \dfrac{R_m}{R_{m_{cr}}} = \dfrac{\delta}{\omega_0}$	$R_{m_{cr}} = 2\omega_0 M_m$
Damping coefficient	$\delta = R_m / 2M_m = \zeta\omega_0$		$\delta_{rez} = R_{m_{cr}} / 2M_m \cong \omega_0\varsigma$
Driving frequency	ω	$\gamma = \omega / \omega_0$	$\gamma_{rez} = \sqrt{1-2\zeta^2}$
Quality factor	$Q = \dfrac{\omega_0}{2\delta} = \omega_0 \dfrac{M_m}{R_m} = \dfrac{1}{R_m}\sqrt{M_m K_m}$		$Q_0 = \dfrac{1}{2\zeta}$
Static displacement	$q_{st} = \phi_0 / K_m = p / \omega_0^2;\ p = \phi_0 / M_m$	ϕ_0 - amplitude of perturbation	$q_{st} = 2\zeta\left(q_1\right)_{\max}$
Generalized mathematical model: $\ddot{q} + 2\delta\,\dot{q} + \omega_0^2 q = p\sin\omega t$; Transitory component: $q_t = q_0 e^{-\delta t}\sin\left(\omega_p + \varphi_0\right)$; Permanent component: $q_p = q_1\sin\left(\omega t - \varphi\right)$			
Characteristic equation	$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0$; solution: $x_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$		
Amplitude	$q_1 = \dfrac{p}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\delta\omega\right)^2}}$	$q_1 = \dfrac{q_{st}}{\sqrt{\left(1-\gamma^2\right)^2 + 4\zeta^2\gamma^2}}$	$\left(q_1\right)_{\max} = \dfrac{q_{st}}{2\zeta\sqrt{1-\zeta^2}} \cong$ $\cong q_{st} / 2\zeta = p / 2\delta\omega_0$
Phase	$\varphi = arctg\dfrac{2\delta\omega}{\omega_0^2 - \omega^2}$	$\varphi = arctg\dfrac{\gamma}{(1-\gamma^2)Q}$	$\varphi_{rez} = arctg\left[\sqrt{\zeta^{-2} - 2}\right]$
Dynamic amplification factor	$\xi = \left\|q_1 / q_{st}\right\| = \left\|1-\gamma^2\right\|^{-1}$	$\xi = \dfrac{1}{\sqrt{(1-\gamma^2)^2 + 4\gamma^2\zeta^2}}$	$\xi_{rez} = \dfrac{1}{2\xi\sqrt{1-\zeta^2}} \cong Q_0$
Logarithmic decrement	$\delta_1 = \ln\left(q_k / q_{k+1}\right)$; $\delta_1 = 2\pi\zeta / \sqrt{1-\zeta^2} \cong 2\pi\zeta\left(\zeta \ll 1\right)$; $\pi / \delta_1 = 1/2\zeta$ - over amplitude coefficient		
Dynamic elastic constant	$\underline{K}_D = \underline{\phi} / \underline{Z} = K_m - M_m\omega^2 + j\omega K_m = j\omega\underline{Z}_m$; $q_1 = \left\|z\right\|; z = x + jy$; $\underline{Z}_m = R_m + jX_m$		
Energy	$W = \dfrac{2\pi^2 p^2\omega^2}{\omega_0^4\left[(1-\gamma^2)^2 + \gamma^2 Q^{-2}\right]}$	$\dfrac{W}{W_{rez}} = \dfrac{\gamma^2}{\left[Q^2(1-\gamma^2) + \gamma^2\right]}$	$W_{rez} = \dfrac{2\pi^2 Q_0}{\omega_0^2}$

Table 6. The basic features of mechanical systems with distributed parameters (for S.D.P.)

Quantities; characteristic constants		Definitions, formulas, mathematical models and their solutions			
		General case (R'; L'; C'; G' ≠ 0)	Without perditance (G' = 0)	Without losses (R' = G' = 0)	Without distortion
Constants	Wave period	$T_0=\omega_0^{-1}=\frac{L'}{R'}=\frac{1}{2\delta_1};\ \omega_0=2\pi f_0$			
Constants	Damping coeff.	$\delta=\frac{R'}{2H'};\ \delta_1=\frac{G'}{2C'};\ \delta_2=\frac{R'}{H'}=2\delta$	$\delta_1=0$	$\delta=\delta_1=0$	$\delta=\delta_1=\frac{\delta_2}{2}$
Time: $\delta_3=\frac{x}{a}\,[s]$; $x=L\Rightarrow T_p=\frac{L}{a}$; wave propagation speed: $a=\frac{1}{\sqrt{H'C'}}$; $a_0=\frac{1}{\sqrt{H'C'_0}}$; wavelength: $\lambda_u=aT_0=\frac{a}{f_0}$					
Propagation factor	Operational	$\gamma(s)=\sqrt{(R'+H's)(G'+C's)}$	$\gamma(s)=\sqrt{(R'+H's)C's}$	$\gamma(s)=s\sqrt{H'C'}=\frac{s}{a}$	$\gamma(s)=s+\frac{\delta_2}{a}$
Propagation factor	Complex	$\gamma(j\omega)=\sqrt{(R'+H'j\omega)(G'+C'j\omega)};\ \gamma=\delta+j\omega$	$\gamma(j\omega)=$ $=\sqrt{-H'C'\omega^2+j\omega R'C'}$	$\gamma(j\omega)=\frac{j\omega}{a}$	
Impedance	Operational	$Z_c(s)=\sqrt{(R'+H's)(G'+C's)}=Z_{c_0}\sqrt{\frac{s+2\delta}{s+2\delta_1}}$	$Z_c=Z_{c_0}\sqrt{1+\frac{2\delta}{s}}$	$Z_c\to Z_{c_0}$	
Impedance	Complex	$\underline{Z}_c(j\omega)=\frac{R'+j\omega H'}{\sqrt{R'G'-C'H'\omega^2+j\omega(R'C'+H'G')}}$; $Z_{c_0}=\sqrt{\frac{H'}{C'}}=\frac{\rho a_0}{s}$ - wave impedance			
Load (terminal) impedance		$\underline{Z}_c=R_s+jx_s;\ \underline{Z}_c=\underline{Z}_s\Rightarrow R_s=\frac{1}{\sqrt{2C'}}\sqrt{H'+\sqrt{(H')^2+\left(\frac{R'}{\omega}\right)^2}}$; $X_s=\frac{1}{\sqrt{2C'}}\sqrt{-H'+\sqrt{(H')^2+\left(\frac{R'}{\omega}\right)^2}}$			
Mathematical model	Coupled eq.	$\frac{\partial(\Delta e)}{\partial x}=-\left[R'\Delta f+H'\frac{\partial(\Delta f)}{\partial t}\right];\ \frac{\partial(\Delta f)}{\partial x}=-\left[R'\Delta e+H'\frac{\partial(\Delta e)}{\partial t}\right]$			
Mathematical model	Decoupled eq.	$\frac{\partial^2(\Delta e)}{\partial x^2}=H'C'\frac{\partial^2(\Delta e)}{\partial t^2}+(R'C'+G'H')\frac{\partial(\Delta e)}{\partial t}+R'G'\Delta e$ $\frac{\partial^2(\Delta f)}{\partial x^2}=H'C'\frac{\partial^2(\Delta f)}{\partial t^2}+(R'C'+G'H')\frac{\partial(\Delta f)}{\partial t}+R'G'\Delta f$			
Systemic model		$\frac{d^2(\Delta e)}{dx^2}-(R'+H's)(G'+C's)\Delta e=0$; $\frac{d^2(\Delta f)}{dx^2}-(R'+H's)(G'+C's)\Delta f=0$			
Solution	General	$\overline{\Delta e}(s,x)=\overline{C_1}e^{\gamma x}+\overline{C_2}e^{-\gamma x}$ $\overline{\Delta f}(s,x)=\overline{C_1}e^{\gamma x}+\overline{C_2}e^{-\gamma x}$			
Solution	Final	$\Delta e(s,x)=\frac{1}{sh(\gamma L)}\left\{\overline{\Delta e}_2 sh(\gamma x)+\overline{\Delta e}_1 sh\left[\gamma(L-x)\right]\right\}$ $\Delta f(s,x)=\frac{1}{sh(\gamma L)}\left\{\overline{\Delta f}_2 sh(\gamma x)+\overline{\Delta f}_1 sh\left[\gamma(L-x)\right]\right\}$			
Matrix input - output complex relationship	Operational	$\begin{bmatrix}\underline{\Delta e}(j\omega)\\ \underline{\Delta f}(j\omega)\end{bmatrix}=\begin{bmatrix}ch(\gamma x) & \underline{Z}_c sh(\gamma x)\\ \frac{1}{\underline{Z}_c}sh(\gamma x) & ch(\gamma x)\end{bmatrix}\begin{bmatrix}\underline{\Delta e_1}(j\omega)\\ \underline{\Delta f_1}(j\omega)\end{bmatrix}$;			
Matrix input - output complex relationship	Complex	$\begin{bmatrix}\underline{\Delta e_2^*}(j\omega)\\ \underline{\Delta f_2^*}(j\omega)\end{bmatrix}=\begin{bmatrix}ch(\gamma(j\omega)L) & [\underline{Z}_c(j\omega)/\underline{Z}_{c_0}(j\omega)]sh(\gamma(j\omega)L)\\ [\underline{Z}_{c_0}(j\omega)/\underline{Z}_c(j\omega)]sh(\gamma(j\omega)L) & ch(\gamma(j\omega)L)\end{bmatrix}\begin{bmatrix}\underline{\Delta e_1^*}(j\omega)\\ \underline{\Delta f_1^*}(j\omega)\end{bmatrix}$			

Table 7. Comparison between a mechanical system, an electric series circuit and an electric parallel circuit

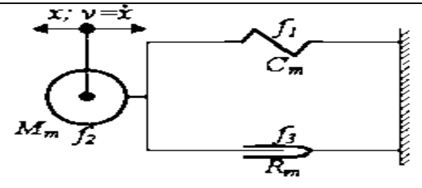
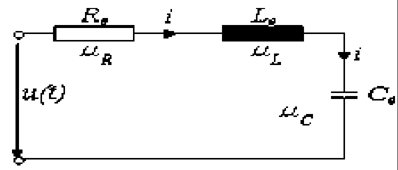
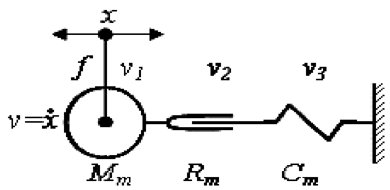
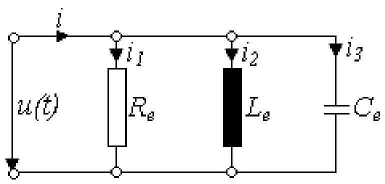
Model of the mechanic system / device and of the analogue electric circuit		Mechanical system / device	Type of analogy	Analogue electric circuit
	Physical		Type I (direct) - in impedance $\underline{Z}_m \leftrightarrow \underline{Z}_e$ $f \leftrightarrow u$; $v \leftrightarrow i$; $R_m \leftrightarrow R_e$; $M_m \leftrightarrow L_e$; $K_m^{-1} = C_m \leftrightarrow C_e$	
	Mathematical	$M_m \dot{v}(t) + R_m v(t) + \frac{1}{C_m}\int_0^t v(\tau)d\tau = f_k$ $v(t) = \frac{F}{Z_m}\cos(\omega t + \varphi)$; $Q_m = \frac{M_m}{R_m}\omega_0 = \frac{1}{\omega_0 R_m C_m}$ $\varphi = arctg\left[\frac{(\omega/\omega_0)^2 - 1}{\omega R_m C_m}\right]$	Type II (inverse) - in admittance $f \leftrightarrow i$; $v \leftrightarrow u$; $R_m \leftrightarrow G_e = R_e^{-1}$; $M_m \leftrightarrow C_e$; $K_m^{-1} = C_m \leftrightarrow L_e$; $\underline{Z}_m \leftrightarrow \underline{Y}_e$	$L_e \dot{i}(t) + R_e i(t) + \frac{1}{C_e}\int_0^t i(\tau)d\tau = u_k$ $i(t) = \frac{U}{Z_e}\cos(\omega t + \varphi)$; $Q_m = \frac{L_e}{R_e}\omega_0 = \frac{1}{\omega_0 R_e C_e}$ $\varphi = arctg\left[\frac{(\omega/\omega_0)^2 - 1}{\omega R_e C_e}\right]$
	Physical		Type I (direct) - in impedance $\underline{Z}_m \leftrightarrow \underline{Z}_e$ $f \leftrightarrow u$; $v \leftrightarrow i$; $R_m \leftrightarrow R_e$; $M_m \leftrightarrow L_e$; $K_m^{-1} = C_m \leftrightarrow C_e$	
	Mathematical	$C_m \overset{\bullet}{f}(t) + G_m f(t) + \frac{1}{M_m}\int_0^t f(\tau)d\tau = v_k$ $f(t) = Z_m V\cos(\omega t + \varphi)$ $\varphi = arctg\left[\frac{(\omega/\omega_0)^2 - 1}{\omega M_m}R_m\right]$	$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$ (impedance) $\underline{Y} = G + j\left(\omega C - \frac{1}{\omega L}\right)$ (admittance) $\omega_0 = (LC)^{-1/2}$ - natural frequency	$C_e \overset{\bullet}{u}(t) + G_e u(t) + \frac{1}{L_e}\int_0^t u(\tau)d\tau = v$ $u(t) = Z_e I\cos(\omega t + \varphi)$ $\varphi = arctg\left[\frac{(\omega/\omega_0)^2 - 1}{\omega L_e}R_e\right]$

Table 8. The mathematical model of an acoustic line with D.P., established using the electroacoustic analogy

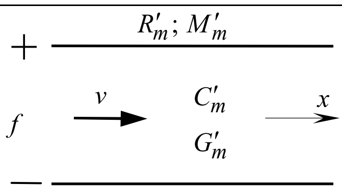
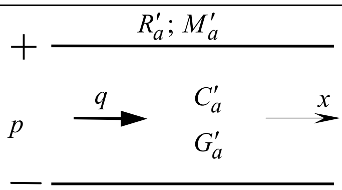
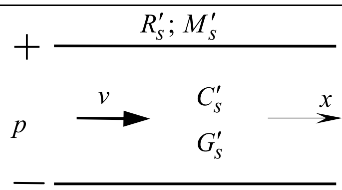
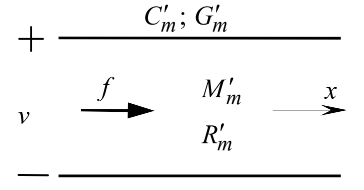
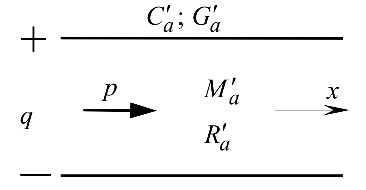
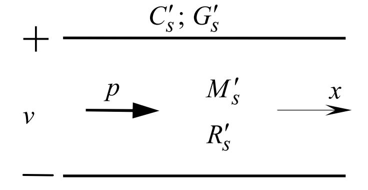
Domain / Type			Type of oscillating - harmonic system		
			Mechanical system with longitudinal motion	Acoustic system with constant section (S=$ct.$)	Uniform one-dimensional acoustic system
Type-I-analogy (in impedance)	Physical model				
	Mathematical model	Equations	$\frac{\partial f}{\partial x} + R_m' v + M_m' \frac{\partial v}{\partial t} = 0$	$\frac{\partial p_a}{\partial x} + R_a' q + M_a' \frac{\partial q}{\partial t} = 0$	$\frac{\partial p}{\partial x} + R_s' v + M_s' \frac{\partial v}{\partial t} = 0$
		System parameters	$M_m' = \rho S$; $C_m' = (ES)^{-1}$ $Z_u = \sqrt{\frac{R_m' + j\omega M_m'}{G_m' + j\omega C_m'}}$ $Z_u^0 = \rho c S$; $c = \sqrt{\frac{E}{\rho}}$	$M_a' = \frac{\rho}{S}$; $C_a' = \frac{S}{E}$; $Z_u = \sqrt{\frac{R_a' + j\omega M_a'}{G_a' + j\omega C_a'}}$ $Z_u^0 = \frac{\rho c}{S}$; $c = \sqrt{\frac{E}{\rho}}$	$M_s' = \rho$; $C_s' = E^{-1}$; $Z_u = \sqrt{\frac{R_s' + j\omega M_s'}{G_s' + j\omega C_s'}}$ $Z_u^0 = \rho c$; $c = \sqrt{\frac{E}{\rho}}$

Table 8. Continuation

Type- II-analogy (in admittance)					
	Physical model				
	Mathematical model	Equations	$\frac{\partial v}{\partial x} + G'_m f + C'_m \frac{\partial f}{\partial t} = 0$	$\frac{\partial q}{\partial x} + G'_a p + C'_a \frac{\partial p}{\partial t} = 0$	$\frac{\partial v}{\partial x} + G'_s p + C'_s \frac{\partial p}{\partial t} = 0$
		System parameters	$M'_m = \rho S;\ C'_m = (ES)^{-1};$ $Z_u = \sqrt{\frac{G'_m + j\omega C'_m}{R'_m + j\omega M'_m}}$ $Z_u^0 = \frac{1}{\rho c S};\ c = \sqrt{\frac{E}{\rho}}$	$M'_a = \frac{\rho}{S};\ C'_a = \frac{S}{E};$ $Z_u = \sqrt{\frac{G'_a + j\omega C'_a}{R'_a + j\omega M'_a}}$ $Z_u^0 = \frac{S}{\rho c};\ c = \sqrt{\frac{E}{\rho}}$	$M'_s = \rho;\ C'_s = E^{-1};$ $Z_u = \sqrt{\frac{G'_s + j\omega C'_s}{R'_s + j\omega M'_s}}$ $Z_u^0 = \frac{1}{\rho c};\ c = \sqrt{\frac{E}{\rho}}$
ρ- density; E - Young's modulus; p - acoustic pressure; q - acoustic flow; v - velocity of the medium					

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